Exam. Code : 103201
Subject Code : 1026

B.A./B.Sc. 1st Semester (Batch 2021-24)
MATHEMATICS
Paper—II
(Calculus & Trigonometry)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt ***five*** questions in all, selecting at least ***one*** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Find the g.l.b. and l.u.b. of the set $\left\{\frac{2x+1}{x+5} : |x-4|<2\right\}$.

(b) Let $f(x) = \begin{cases} 1, & x \le 3 \\ ax+b, & 3<x<5 \\ 7, & x \ge 5 \end{cases}$

Determine the constants a and b so that f may be continuous for all x. 5,5

2. (a) Prove that the union of two bounded sets is a bounded set.

(b) Show that $f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}$, $x \neq 0$ and $f(0) = 0$ is discontinuous at $x = 0$. Also locate the type of discontinuity. 5,5

SECTION—B

3. (a) Prove that $\frac{d}{dx}\{\tanh(\log x)\} = \frac{4x}{(x^2+1)^2}$.

(b) Show that $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = e^{-\frac{1}{6}}$ 5,5

4. (a) If $y = e^m \cos^{-1} x$, then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$.

(b) State and prove Taylor's theorem with Lagrange's form of remainder after n terms. 5,5

SECTION—C

5. (a) If $x + \frac{1}{x} = 2\cos\theta$, prove that $x^n + \frac{1}{x^n} = 2\cos n\theta$ and $x^n - \frac{1}{x^n} = \pm 2i \sin n\theta$, where n is a + ve integer.

(b) If $\cos(\theta + i\phi) = \cos\alpha + i\sin\alpha$, then show that $\cos 2\theta + \cosh 2\phi = 2$. 5,5

6. (a) If $A + iB = C\tan(x+iy)$, show that $\tan 2x = \frac{2CA}{C^2 - A^2 - B^2}$.

(b) If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$, n being a + ve integer. 5,5

SECTION—D

7. (a) Prove that $\log(1 + i\tan\alpha) = \log\sec\alpha + i\alpha$, α being a + ve acute angle.

(b) Expand $\sin 6\theta$ in powers of $\sin\theta$ and $\cos\theta$ and deduce that

$$\frac{\sin 6\theta}{\sin\theta} = 6\cos\theta - 32\cos^3\theta + 32\cos^5\theta.$$

5,5

8. (a) Use Gregory's series to prove that :

$$\left(\frac{1}{2}+\frac{1}{3}\right) - \frac{1}{3}\left(\frac{1}{2^3}+\frac{1}{3^3}\right) + \frac{1}{5}\left(\frac{1}{2^5}+\frac{1}{3^5}\right) + \ldots\ldots = \frac{\pi}{4}.$$

(b) Sum to n terms of the series $\tan\alpha\tan 2\alpha + \tan 2\alpha\tan 3\alpha + \tan 3\alpha\tan 4\alpha + \ldots\ldots$ 5,5